Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C4

Advanced Level

Wednesday 25 January 2012 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

P40085A

1. The curve C has the equation $2x + 3y^2 + 3x^2 y = 4x^2$.

The point P on the curve has coordinates $(-1, 1)$.

(*a*) Find the gradient of the curve at P.

(5)

(*b*) Hence find the equation of the normal to C at P, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(3)

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2. (*a*) Use integration by parts to find $\int x \sin 3x \;\; \mathrm{d}x$.

(3)

(*b*) Using your answer to part (*a*), find $\int x^2 \cos 3x \;\; \mathrm{d}x$.

(3)

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3. (*a*) Expand

$$\frac{1}{(2-5x)^2}, \qquad |x| < \frac{2}{5},$$

in ascending powers of x, up to and including the term in x^2, giving each term as a simplified fraction.

(5)

Given that the binomial expansion of $\dfrac{2+kx}{(2-5x)^2}$, $|x| < \dfrac{2}{5}$, is

$$\frac{1}{2} + \frac{7}{4}x + Ax^2 + \ldots,$$

(*b*) find the value of the constant k,

(2)

(*c*) find the value of the constant A.

(2)

4.

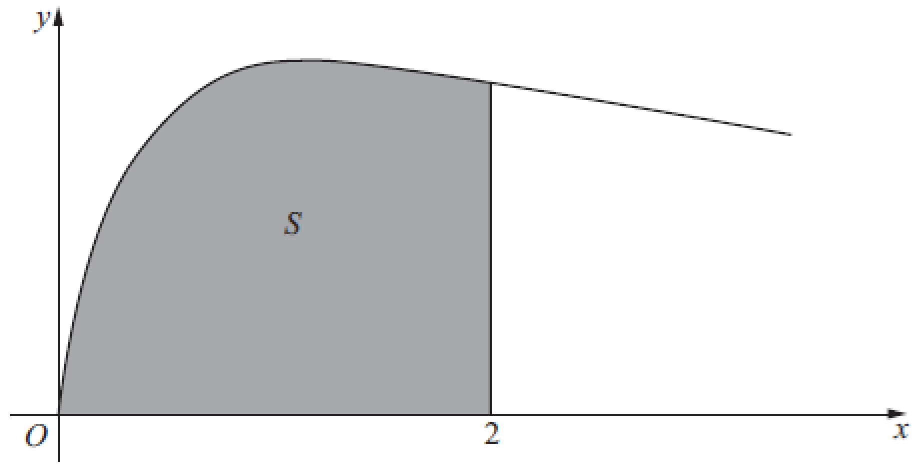


Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0.$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$.

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

5.

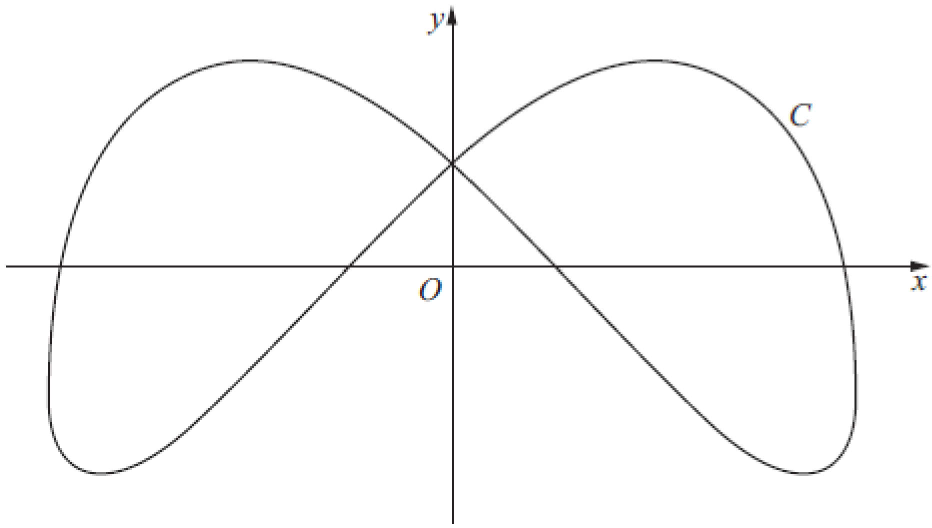


Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 4\sin\left(t + \frac{\pi}{6}\right), \qquad y = 3\cos 2t, \qquad 0 \leq t < 2\pi.$$

(a) Find an expression for $\frac{dy}{dx}$ in terms of t.

(3)

(b) Find the coordinates of all the points on C where $\frac{dy}{dx} = 0$.

(5)

6.

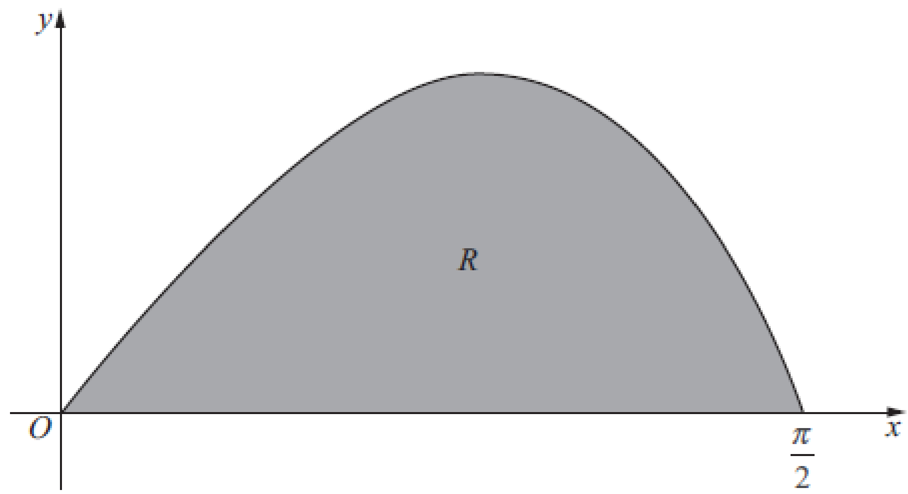


Figure 3

Figure 3 shows a sketch of the curve with equation $y = \dfrac{2\sin 2x}{(1+\cos x)}$, $0 \leq x \leq \dfrac{\pi}{2}$.

The finite region R, shown shaded in Figure 3, is bounded by the curve and the x-axis.

The table below shows corresponding values of x and y for $y = \dfrac{2\sin 2x}{(1+\cos x)}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y	0		1.17157	1.02280	0

(*a*) Complete the table above giving the missing value of y to 5 decimal places.

(1)

(*b*) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate for the area of R, giving your answer to 4 decimal places.

(3)

(*c*) Using the substitution $u = 1 + \cos x$, or otherwise, show that

$$\int \frac{2\sin 2x}{(1+\cos x)}\,\mathrm{d}x = 4\ln(1+\cos x) - 4\cos x + k,$$

where k is a constant.

(5)

(*d*) Hence calculate the error of the estimate in part (*b*), giving your answer to 2 significant figures.

(3)

7. Relative to a fixed origin O, the point A has position vector $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$,

the point B has position vector $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$,

and the point D has position vector $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$.

(2)

(*b*) Find a vector equation for the line l.

(2)

(*c*) Show that the size of the angle BAD is 109°, to the nearest degree.

(4)

The points A, B and D, together with a point C, are the vertices of the parallelogram $ABCD$, where $\overrightarrow{AB} = \overrightarrow{DC}$.

(*d*) Find the position vector of C.

(2)

(*e*) Find the area of the parallelogram $ABCD$, giving your answer to 3 significant figures.

(3)

(*f*) Find the shortest distance from the point D to the line l, giving your answer to 3 significant figures.

(2)

8. (*a*) Express $\dfrac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{1}{15}P(5-P), \qquad t \geq 0,$$

where P, in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

(*b*) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + c\mathrm{e}^{-\frac{1}{3}t}}$$

where a, b and c are integers.

(8)

(*c*) Hence show that the population cannot exceed 5000.

(1)

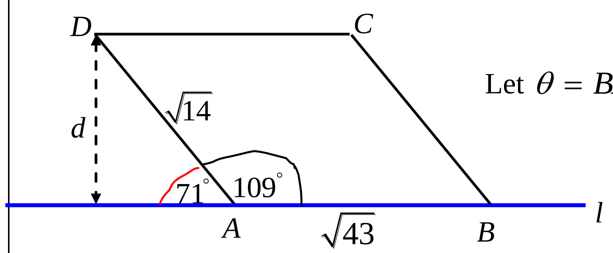
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\left\{\bcancel{\frac{\mathrm{d}y}{\mathrm{d}x}}\ \bcancel{\times}\right\}\quad \underline{2+6y\frac{\mathrm{d}y}{\mathrm{d}x}}+\left(\underline{\underline{6xy+3x^2\frac{\mathrm{d}y}{\mathrm{d}x}}}\right)=\underline{8x}$	M1 A1 B1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8x-2-6xy}{6y+3x^2}\right\}$ *not necessarily required.*	
	At $P(-1,1)$, $\quad \mathrm{m}(\mathbf{T})=\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8(-1)-2-6(-1)(1)}{6(1)+3(-1)^2}=-\frac{4}{9}$	dM1 A1 **cso**
		[5]
(b)	So, $\mathrm{m}(\mathbf{N})=\frac{-1}{-\frac{4}{9}}\ \left\{=\frac{9}{4}\right\}$	M1
	$\mathbf{N}$: $y-1=\frac{9}{4}(x+1)$	M1
	$\mathbf{N}$: $9x-4y+13=0$	A1
		[3]
		(8 marks)
2. (a)	$\int x\sin 3x\,\mathrm{d}x=-\frac{1}{3}x\cos 3x-\int -\frac{1}{3}\cos 3x\,\{\mathrm{d}x\}$	M1 A1
	$=-\frac{1}{3}x\cos 3x+\frac{1}{9}\sin 3x\ \{+c\}$	A1
		[3]
(b)	$\int x^2\cos 3x\,\mathrm{d}x=\frac{1}{3}x^2\sin 3x-\int \frac{2}{3}x\sin 3x\,\{\mathrm{d}x\}$	M1 A1
	$=\frac{1}{3}x^2\sin 3x-\frac{2}{3}\left(-\frac{1}{3}x\cos 3x+\frac{1}{9}\sin 3x\right)\ \{+c\}$	A1 **isw**
	$\left\{=\frac{1}{3}x^2\sin 3x+\frac{2}{9}x\cos 3x-\frac{2}{27}\sin 3x\ \{+c\}\right\}$ *Ignore subsequent working*	**[3]**
		(6 marks)

Question Number	Scheme		Marks
3. (a)	$\frac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}}\left(1-\frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}}\left(1-\frac{5x}{2}\right)^{-2}$	$\underline{(2)^{-2}}$ or $\underline{\frac{1}{4}}$	B1
	$= \left\{\frac{1}{4}\right\}\left[1+(-2)(**x)+\frac{(-2)(-3)}{2!}(**x)^2+\ldots\right]$		M1 A1ft
	$= \left\{\frac{1}{4}\right\}\left[\underline{1+(-2)\left(-\frac{5x}{2}\right)+\frac{(-2)(-3)}{2!}\left(-\frac{5x}{2}\right)^2+\ldots}\right]$		
	$= \frac{1}{4}\left[1+5x;+\frac{75}{4}x^2+\ldots\right]$		
	$= \frac{1}{4}+\frac{5}{4}x;+\frac{75}{16}x^2+\ldots$		A1; A1
			[5]
(b)	$\left\{\frac{2+kx}{(2-5x)^2}\right\} = (2+kx)\left(\frac{1}{4}+\frac{5}{4}x+\left\{\frac{75}{16}x^2+\ldots\right\}\right)$	*Can be implied by later work even in part (c).*	M1
	x terms: $\frac{2(5x)}{4}+\frac{kx}{4}=\frac{7x}{4}$		
	giving, $10+k=7 \Rightarrow \underline{k=-3}$	$\underline{k=-3}$	A1
			[2]
(c)	x^2 terms: $\frac{150x^2}{16}+\frac{5kx^2}{4}$		M1
	So, $A = \frac{75}{8}+\frac{5(-3)}{4} = \frac{75}{8}-\frac{15}{4} = \underline{\frac{45}{8}}$	$\underline{\frac{45}{8}}$ or $\underline{5\frac{5}{8}}$ or $\underline{5.625}$	A1
			[2] **(9 marks)**
4.	Volume $= \underline{\pi\int_0^2\left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2 \mathrm{d}x}$	Use of $V=\underline{\pi\int y^2}\,\mathrm{d}x$.	B1
	$= (\pi)\left[\frac{1}{3}\ln\left(3x^2+4\right)\right]_0^2$	$\pm k\ln\left(3x^2+4\right)$	M1
		$\frac{1}{3}\ln\left(3x^2+4\right)$	A1
	$= (\pi)\left[\left(\frac{1}{3}\ln 16\right)-\left(\frac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $= \underline{\frac{1}{3}\pi\ln 4}$	$\underline{\frac{1}{3}\pi\ln 4}$ or $\underline{\frac{2}{3}\pi\ln 2}$	A1 oe isw
			[5] **(5 marks)**

Question Number	Scheme	Marks
5.	$x = 4\sin\left(t + \frac{\pi}{6}\right), \quad y = 3\cos 2t, \quad 0 \leqslant t < 2\pi$	
(a)	$\frac{dx}{dt} = 4\cos\left(t + \frac{\pi}{6}\right), \quad \frac{dy}{dt} = -6\sin 2t$	B1 B1
	So, $\frac{dy}{dx} = \frac{-6\sin 2t}{4\cos\left(t + \frac{\pi}{6}\right)}$	B1$\sqrt{}$ oe (3)
(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\} -6\sin 2t = 0$	M1 oe
	@ $t = 0, \quad x = 4\sin\left(\frac{\pi}{6}\right) = 2, \quad y = 3\cos 0 = 3 \quad \to (2, 3)$	M1
	@ $t = \frac{\pi}{2}, \quad x = 4\sin\left(\frac{2\pi}{3}\right) = \frac{4\sqrt{3}}{2}, \quad y = 3\cos\pi = -3 \to (2\sqrt{3}, -3)$	
	@ $t = \pi, \quad x = 4\sin\left(\frac{7\pi}{6}\right) = -2, \quad y = 3\cos 2\pi = 3 \to (-2, 3)$	
	@ $t = \frac{3\pi}{2}, x = 4\sin\left(\frac{5\pi}{3}\right) = \frac{4(-\sqrt{3})}{2}, \quad y = 3\cos 3\pi = -3 \to (-2\sqrt{3}, -3)$	A1 A1 A1 (5)
		(8 marks)
6. (a)	0.73508	B1 **cao** (1)
(b)	Area $\approx \frac{1}{2} \times \frac{\pi}{8}; \times \left[0 + 2(\text{their } 0.73508 + 1.17157 + 1.02280) + 0\right]$	B1 M1
	$= \frac{\pi}{16} \times 5.8589... = 1.150392325... = 1.1504$ (4 dp) awrt 1.1504	A1 (3)
(c)	$\{u = 1 + \cos x\} \Rightarrow \frac{du}{dx} = -\sin x$	B1
	$\left\{\int \frac{2\sin 2x}{(1 + \cos x)}\, dx = \right\} \int \frac{2(2\sin x\cos x)}{(1 + \cos x)}\, dx$ $\sin 2x = 2\sin x\cos x$	B1
	$= \int \frac{4(u-1)}{u}.(-1)\, du \quad \left\{= 4\int \frac{(1-u)}{u}\, du\right\}$	M1
	$= 4\int \left(\frac{1}{u} - 1\right) du = 4(\ln u - u) + c$	dM1
	$= 4\ln(1 + \cos x) - 4(1 + \cos x) + c \quad = 4\ln(1 + \cos x) - 4\cos x + k$ **AG**	A1 cso (5)
(d)	$= \left[4\ln\left(1 + \cos\frac{\pi}{2}\right) - 4\cos\frac{\pi}{2}\right] - \left[4\ln(1 + \cos 0) - 4\cos 0\right]$ Applying limits $x = \frac{\pi}{2}$ and $x = 0$ either way round.	M1
	$= [4\ln 1 - 0] - [4\ln 2 - 4]$	
	$= 4 - 4\ln 2 \; \{= 1.227411278...\}$ $\pm 4(1 - \ln 2)$ or $\pm(4 - 4\ln 2)$ or awrt ± 1.2	A1
	Error $= \left\|(4 - 4\ln 2) - 1.1504...\right\|$ $= 0.0770112776... = 0.077$ (2sf) awrt ± 0.077 or awrt $\pm 6.3(\%)$	A1 cso (3)
		(12 marks)

Question Number	Scheme	Marks
7.	$\overrightarrow{OA} = 2\mathbf{i} - \mathbf{j} + 5\mathbf{k}\ ,\ \overrightarrow{OB} = 5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k},\ \{\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}\}\ \ \&\ \ \overrightarrow{OD} = -\mathbf{i} + \mathbf{j} + 4\mathbf{k}$	
(a)	$\overrightarrow{AB} = = \pm\left((5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) - (2\mathbf{i} - \mathbf{j} + 5\mathbf{k})\right); = 3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$	M1; A1 **[2]**
(b)	$l:\ \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 10 \end{pmatrix} + \lambda\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix}$	M1 A1ft **[2]**
	 Let $\theta = B\hat{A}D$ Let d be the shortest distance from C to l.	
(c)	$\overrightarrow{AD} = \overrightarrow{OD} - \overrightarrow{OA} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} = \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}$ or $\overrightarrow{DA} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$	M1
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \overrightarrow{AD}}{\lvert\overrightarrow{AB}\rvert.\lvert\overrightarrow{AD}\rvert} = \dfrac{\begin{pmatrix} 3 \\ 3 \\ 5 \end{pmatrix} \bullet \begin{pmatrix} -3 \\ 2 \\ -1 \end{pmatrix}}{\sqrt{(3)^2 + (3)^2 + (5)^2}\,.\sqrt{(-3)^2 + (2)^2 + (-1)^2}}$ Applies dot product formula between their $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ and their $\left(\overrightarrow{AD}\text{ or }\overrightarrow{DA}\right)$.	M1
	$\cos\theta = \pm\left(\dfrac{-9 + 6 - 5}{\sqrt{(3)^2 + (3)^2 + (5)^2}\,.\sqrt{(-3)^2 + (2)^2 + (-1)^2}}\right)$ Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{-8}{\sqrt{43}.\sqrt{14}} \Rightarrow \theta = 109.029544... = 109$ (nearest °) awrt 109	A1 **cso AG** **[4]**
(d)	$\overrightarrow{OC} = \overrightarrow{OD} + \overrightarrow{DC} = \overrightarrow{OD} + \overrightarrow{AB} = (-\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + (3\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OB} + \overrightarrow{AD} = (5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k}) + (-3\mathbf{i} + 2\mathbf{j} - \mathbf{k})$	M1
	So, $\overrightarrow{OC} = 2\mathbf{i} + 4\mathbf{j} + 9\mathbf{k}$	A1 **[2]**
(e)	Area $ABCD = \left(\tfrac{1}{2}(\sqrt{43})(\sqrt{14})\sin 109^\circ\right); \times 2 = 23.19894905$ awrt 23.2	M1; dM1 A1 **[3]**
(f)	$\dfrac{d}{\sqrt{14}} = \sin 71$ or $\sqrt{43}\,d = 23.19894905...$	M1
	$\therefore\ d = \sqrt{14}\sin 71^\circ = 3.537806563...$ awrt 3.54	A1 **[2]** **(15 marks)**

Question Number	Scheme		Marks
8. (a)	$1 = A(5-P) + BP$	Can be implied.	M1
	$A = \frac{1}{5},\ B = \frac{1}{5}$	Either one.	A1
	giving $\frac{\frac{1}{5}}{P} + \frac{\frac{1}{5}}{(5-P)}$		A1 **cao, aef**
			[3]
(b)	$\int \frac{1}{P(5-P)}\,dP = \int \frac{1}{15}\,dt$		B1
	$\frac{1}{5}\ln P - \frac{1}{5}\ln(5-P) = \frac{1}{15}t\ \ (+c)$		M1* A1ft
	$\{t=0,\ P=1 \Rightarrow\}\quad \frac{1}{5}\ln 1 - \frac{1}{5}\ln(4) = 0 + c \quad \left\{\Rightarrow\ c = -\frac{1}{5}\ln 4\right\}$		dM1*
	eg: $\frac{1}{5}\ln\left(\frac{P}{5-P}\right) = \frac{1}{15}t - \frac{1}{5}\ln 4$	Using any of the subtraction (or addition) laws for logarithms CORRECTLY	dM1*
	$\ln\left(\frac{4P}{5-P}\right) = \frac{1}{3}t$		
	eg: $\frac{4P}{5-P} = e^{\frac{1}{3}t}$ or eg: $\frac{5-P}{4P} = e^{-\frac{1}{3}t}$	Eliminate ln's correctly.	dM1*
	gives $4P = 5e^{\frac{1}{3}t} - Pe^{\frac{1}{3}t} \Rightarrow P(4 + e^{\frac{1}{3}t}) = 5e^{\frac{1}{3}t}$		
	$P = \frac{5e^{\frac{1}{3}t}}{(4 + e^{\frac{1}{3}t})}\quad \left\{\frac{(\div e^{\frac{1}{3}t})}{(\div e^{\frac{1}{3}t})}\right\}$	Make P the subject.	dM1*
	$P = \frac{5}{(1 + 4e^{-\frac{1}{3}t})}$ or $P = \frac{25}{(5 + 20e^{-\frac{1}{3}t})}$ etc.		A1
			[8]
(c)	$1 + 4e^{-\frac{1}{3}t} > 1 \Rightarrow P < 5$. So population cannot exceed 5000.		B1
			[1] **(12 marks)**